Unofficial Solutions of “Solar Energy” Tutorial 6 (WT2021/22)

This is a rewrite of the sample solution (that is not handed out to the students) taken from a hybrid tutorial.

Exercise 1

a. False

b. True

c. False

d. False

Exercise 2

The Lambert-Beer law describes how the intensity of the light beam decays as it propagates through an absorbing medium with a certain absorption coefficient $\alpha(\lambda)$:

$$I(d) = I_0 \cdot \exp(-\alpha(\lambda) \cdot d)$$

First the absoption coeffient is taken from the figure, after converting the energy of the photons into wavelenght via:

$$\lambda(\text{nm}) = \frac{h \cdot c}{q \cdot E_{\text{ph}}} = \frac{6{,}626 \cdot 10^{-34}\,\text{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}{1{,}602 \cdot 10^{-19}\,\text{C} \cdot 1{,}55\,\text{eV}} = 800\,\text{nm}$$

Taking into account that the y-axis is plotted in a logarithmic scale, the absoption coefficient can be estimated and the required thickness of each material can be calculated:

GaAs: $\alpha(800\,\text{nm}) = 1{,}5 \cdot 10^4\,\frac{1}{\text{cm}}$, $\quad d = -\frac{\ln(0{,}1)}{1{,}5 \cdot 10^4\,\frac{1}{\text{cm}}} = 1{,}5\,\mu\text{m}$

InP: $\alpha(800\,\text{nm}) = 4 \cdot 10^4\,\frac{1}{\text{cm}}$, $\quad d = -\frac{\ln(0{,}1)}{4 \cdot 10^4\,\frac{1}{\text{cm}}} = 0{,}58\,\mu\text{m}$

GaAs: $\alpha(800\,\text{nm}) = 6 \cdot 10^4\,\frac{1}{\text{cm}}$, $\quad d = -\frac{\ln(0{,}1)}{6 \cdot 10^4\,\frac{1}{\text{cm}}} = 0{,}38\,\mu\text{m}$

GaAs: $\alpha(800\,\text{nm}) = 9 \cdot 10^2\,\frac{1}{\text{cm}}$, $\quad d = -\frac{\ln(0{,}1)}{9 \cdot 10^2\,\frac{1}{\text{cm}}} = 25{,}6\,\mu\text{m}$

Exercise 3

a. Answer: $E_G = 1{,}9\,\text{eV}$

To calculate the bandgaps, we should take into account the wavelength until which a certain junction absorbs light.

Junction A absorbs light (has a non-zero EQE) until $\lambda = 650\,\text{nm}$. The wavelength can be converted into energy by using the relation:

$$E_G = \frac{h \cdot c}{\lambda \cdot q} = \frac{6{,}626 \cdot 10^{-34}\,\text{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}{650 \cdot 10^{-9}\,\text{m} \cdot 1{,}602 \cdot 10^{-19}\,\text{C}} = 1{,}9\,\text{eV}$$

b. Answer: $E_G = 1{,}46\,\text{eV}$

Junction B absorbs light (has a non-zero EQE) until $\lambda = 850\,\text{nm}$. The wavelength can be converted into energy by using the relation:

$$E_G = \frac{h \cdot c}{\lambda \cdot q} = \frac{6{,}626 \cdot 10^{-34}\,\text{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}{850 \cdot 10^{-9}\,\text{m} \cdot 1{,}602 \cdot 10^{-19}\,\text{C}} = 1{,}46\,\text{eV}$$

c. Answer: $E_G = 0{,}99\,\text{eV}$

Junction C absorbs light (has a non-zero EQE) until $\lambda = 1250\,\text{nm}$. The wavelength can be converted into energy by using the relation:

$$E_G = \frac{h \cdot c}{\lambda \cdot q} = \frac{6{,}626 \cdot 10^{-34}\,\text{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}{1250 \cdot 10^{-9}\,\text{m} \cdot 1{,}602 \cdot 10^{-19}\,\text{C}} = 0{,}99\,\text{eV}$$

d. Answer: C

In a multi-junction solar cell, the cell with the highest band gap is placed at the top, and the cell with the lowest band gap is placed at the bottom. Therefore, junction A acts as the top cell, junction B as the middle cell, and junction C as the bottom cell.

e. Answer: $J_{SC_A} = 10{,}42\,\frac{\text{mA}}{\text{cm}^2}$, $J_{SC_B} = 12{,}10\,\frac{\text{mA}}{\text{cm}^2}$, $J_{SC_C} = 17{,}92\,\frac{\text{mA}}{\text{cm}^2}$

The short circuit current density can be calculated as:

$$J_{SC} = q \cdot \int \text{EQE}(\lambda) \cdot \Phi_{ph_\lambda}\,d\lambda = q \cdot \text{EQE} \cdot \Phi_{ph}$$

Junction A has an EQE of $0{,}7$ between a wavelength of $300\,\text{nm}$ and $650\,\text{nm}$. This gives:

$$J_{SC_A} = 1{,}602 \cdot 10^{-19}\,\text{C} \cdot 0{,}7 \cdot 9{,}3 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}} = 104{,}2\,\frac{\text{A}}{\text{m}^2} = 10{,}42\,\frac{\text{mA}}{\text{cm}^2}$$

Junction B has an EQE of $0{,}9$ between a wavelength of $650\,\text{nm}$ and $850\,\text{nm}$. This gives:

$$J_{SC_B} = 1{,}602 \cdot 10^{-19}\,\text{C} \cdot 0{,}9 \cdot 8{,}4 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}} = 121{,}0\,\frac{\text{A}}{\text{m}^2} = 12{,}10\,\frac{\text{mA}}{\text{cm}^2}$$

Junction C has an EQE of $0{,}8$ between a wavelength of $850\,\text{nm}$ and $1250\,\text{nm}$. This gives:

$$J_{SC_C} = 1{,}602 \cdot 10^{-19}\,\text{C} \cdot 0{,}8 \cdot 1{,}4 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}} = 179{,}2\,\frac{\text{A}}{\text{m}^2} = 17{,}92\,\frac{\text{mA}}{\text{cm}^2}$$

f. Answer: $\eta = 17\,\%$

The efficiency is calculated as:

$$\eta = \frac{J_{SC} \cdot V_{OC} \cdot FF}{P_{in}}$$

The open circuit voltages are:

$$V_{OC_A} = \frac{1{,}9\,\text{eV}}{2q} = 0{,}95\,\text{V}\,, \quad V_{OC_B} = \frac{1{,}46\,\text{eV}}{2q} = 0{,}73\,\text{V}\,, \quad V_{OC_C} = \frac{0{,}99\,\text{eV}}{2q} = 0{,}495\,\text{V}$$

Since the cells are connected in series, the short circuit current density of the multi-junction cell will be limited by the lowest short circuit density. On the other hand, the open circuit voltages will add up. Thus:

$$\begin{aligned}\eta &= \frac{J_{SC} \cdot V_{OC} \cdot FF}{P_{in}} = \frac{J_{SC_A} \cdot \left(V_{OC_A} + V_{OC_B} + V_{OC_C}\right) \cdot FF}{P_{in}} \\ &= \frac{10{,}42\,\frac{\text{mA}}{\text{cm}^2} \cdot \left(0{,}95\,\text{V} + 0{,}73\,\text{V} + 0{,}495\,\text{V}\right) \cdot 0{,}75}{100\,\frac{\text{mW}}{\text{cm}^2}} = 0{,}1966 \mathrel{\hat{=}} 17\,\%\end{aligned}$$

Exercise 4

a. Answer: spectral range A

The bandgap is a measure of the lowest photon energy that can be absorbed in the material. The corresponding wavelength is calculated as:

$$\lambda = \frac{h \cdot c}{q \cdot E_\mathrm{G}} = \frac{6{,}626 \cdot 10^{-34}\,\mathrm{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}}{1{,}602 \cdot 10^{-19}\,\mathrm{C} \cdot 2{,}0\,\mathrm{eV}} = 620\,\mathrm{nm}$$

This corresponds to spectral range A.

b. Answer: $J_\mathrm{SC} = 10{,}4\,\frac{\mathrm{mA}}{\mathrm{cm}^2}$

The short circuit current density can be calculated as:

$$J_\mathrm{SC} = q \cdot \int \mathrm{EQE}(\lambda) \cdot \Phi_{\mathrm{ph}_\lambda}\, \mathrm{d}\lambda = q \cdot \mathrm{EQE} \cdot \Phi_\mathrm{ph}$$

The solar cell has an EQE of $0{,}65$ between a wavelength of $0\,\mathrm{nm}$ and $620\,\mathrm{nm}$. This gives:

$$J_\mathrm{SC} = 1{,}602 \cdot 10^{-19}\,\mathrm{C} \cdot 0{,}65 \cdot 10 \cdot 10^{20}\,\frac{1}{\mathrm{m}^2 \cdot \mathrm{s}} = 104\,\frac{\mathrm{A}}{\mathrm{m}^2} = 10{,}4\,\frac{\mathrm{mA}}{\mathrm{cm}^2}$$

c. Answer: $\eta = 8{,}3\%$

The efficiency is calculated as:

$$\eta = \frac{J_\mathrm{SC} \cdot V_\mathrm{OC} \cdot FF}{P_\mathrm{in}}$$

The open circuit voltage is calculated with the given equation:

$$V_{\mathrm{OC}_\mathrm{A}} = \frac{2\,\mathrm{eV}}{2q} = 1\,\mathrm{V}\,, \quad V_{\mathrm{OC}_\mathrm{B}} = \frac{1{,}46\,\mathrm{eV}}{2q} = 0{,}73\,\mathrm{V}\,, \quad V_{\mathrm{OC}_\mathrm{C}} = \frac{0{,}99\,\mathrm{eV}}{2q} = 0{,}495\,\mathrm{V}$$

Therefore, the efficiency is:

$$\eta = \frac{J_\mathrm{SC} \cdot V_\mathrm{OC} \cdot FF}{P_\mathrm{in}} = \frac{10{,}4\,\frac{\mathrm{mA}}{\mathrm{cm}^2} \cdot 1\,\mathrm{V} \cdot 0{,}8}{100\,\frac{\mathrm{mW}}{\mathrm{cm}^2}} = 0{,}083 \mathrel{\hat{=}} 8{,}3\%$$

d. Answer: spectral range B

In order for the photon emitted by the upconverter to be absorbed in the a-SiC:H layer, the sum of the energy of the two (lower-energy) photons must be at least equal to the a-SiC:H bandgap. Therefore, the minimum energy that each of the two photons must carry is:

$$\frac{2\,\mathrm{eV}}{2\,\mathrm{photons}} = 1\,\frac{\mathrm{eV}}{\mathrm{ph}}$$

$$\lambda = \frac{h \cdot c}{q \cdot E_\mathrm{G}} = \frac{6{,}626 \cdot 10^{-34}\,\mathrm{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}}{1{,}602 \cdot 10^{-19}\,\mathrm{C} \cdot 1\,\mathrm{eV}} = 1240\,\mathrm{nm}$$

Since we assume that photons up to $620\,\mathrm{nm}$ are absorbed in the a-SiC:H layer, the photons can be up-converted in spectral range B.

e. Answer: $J_{SC} = 16\,\frac{\text{mA}}{\text{cm}^2}$, $\quad \eta = 10{,}4\,\%$

The short circuit current density can be calculated as:

$$J_{SC} = q \cdot \int \text{EQE}(\lambda) \cdot \Phi_{\text{ph}_\lambda}\, \text{d}\lambda = q \cdot \text{EQE} \cdot \left(1 \cdot \Phi_{\text{ph}_A} + \frac{1}{2} \cdot \Phi_{\text{ph}_B}\right)$$

Take into account that the factor $\frac{1}{2}$ in front of the second integral is due to the fact that the upconverter converts two low-energy photons into one high-energy photon (we lose half of the photon flux).

The solar cell has an EQE of $0{,}65$ between a wavelength of $0\,\text{nm}$ and $1240\,\text{nm}$. This gives:

$$J_{SC} = 1{,}602 \cdot 10^{-19}\,\text{C} \cdot 0{,}65 \cdot \left(10 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}} + \frac{1}{2} \cdot 11 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}}\right) = 160\,\frac{\text{A}}{\text{m}^2} = 16\,\frac{\text{mA}}{\text{cm}^2}$$

The efficiency is calculated as:

$$\eta = \frac{J_{SC} \cdot V_{OC} \cdot FF}{P_{in}} = \frac{16\,\frac{\text{mA}}{\text{cm}^2} \cdot 1\,\text{V} \cdot 0{,}65}{100\,\frac{\text{mW}}{\text{cm}^2}} = 0{,}104 \mathrel{\hat{=}} 10{,}4\,\%$$

f. Answer: spectral range C

In order for the photon emitted by the upconverter to be absorbed in the a-SiC:H layer, the sum of the energy of the tree (lower-energy) photons must be at least equal to the a-SiC:H bandgap. Therefore, the minimum energy that each of the two photons must carry is:

$$\frac{2\,\text{eV}}{3\,\text{photons}} = 0{,}667\,\frac{\text{eV}}{\text{ph}}$$

$$\lambda = \frac{h \cdot c}{q \cdot E_G} = \frac{6{,}626 \cdot 10^{-34}\,\text{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}{1{,}602 \cdot 10^{-19}\,\text{C} \cdot 0{,}667\,\text{eV}} = 1860\,\text{nm}$$

Since we assume that photons up to $620\,\text{nm}$ are absorbed in the a-SiC:H layer and that photons up to $1240\,\text{nm}$ are converted in upconverter 1, the photons can be up-converted in spectral range C.

g. Answer: $J_{SC} = 18\,\frac{\text{mA}}{\text{cm}^2}$, $\quad \eta = 11{,}7\,\%$

The short circuit current density can be calculated as:

$$J_{SC} = q \cdot \int \text{EQE}(\lambda) \cdot \Phi_{\text{ph}_\lambda}\, \text{d}\lambda = q \cdot \text{EQE} \cdot \left(1 \cdot \Phi_{\text{ph}_A} + \frac{1}{2} \cdot \Phi_{\text{ph}_B} + \frac{1}{3} \cdot \Phi_{\text{ph}_C}\right)$$

Take into account that the factor $\frac{1}{3}$ in front of the third integral is due to the fact that the upconverter converts tree low-energy photons into one high-energy photon (we lose two-third of the photon flux).

The solar cell has an EQE of $0{,}65$ between a wavelength of $0\,\text{nm}$ and $1860\,\text{nm}$. This gives:

$$J_{SC} = 1{,}602 \cdot 10^{-19}\,\text{C} \cdot 0{,}65 \cdot \left(10 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}} + \frac{1}{2} \cdot 11 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}} + \frac{1}{3} \cdot 6 \cdot 10^{20}\,\frac{1}{\text{m}^2 \cdot \text{s}}\right) = 180\,\frac{\text{A}}{\text{m}^2} = 18\,\frac{\text{mA}}{\text{cm}^2}$$

The efficiency is calculated as:

$$\eta = \frac{J_{SC} \cdot V_{OC} \cdot FF}{P_{in}} = \frac{18\,\frac{\text{mA}}{\text{cm}^2} \cdot 1\,\text{V} \cdot 0{,}65}{100\,\frac{\text{mW}}{\text{cm}^2}} = 0{,}117 \mathrel{\hat{=}} 11{,}7\,\%$$